Maths 4th ESO

First steps into CLIL

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Generació Plurilingüe
First Year
2017-2018

Generalitat de Catalunya
Departament d'Ensenyament

MARC PER AL PLURILINGÜISME

INDEX

TEACHING NOTES

CONTEST OF MEASURES

Level: 1st - 4th ESO

Time: 2h

Groups: 4-5 students

Needed material:

Video camera, measuring tape, containers, weighing scales, protractor, chronometer, sand, bags, scissors, string, water, paper and pen.

Sequence:
The contest will be in the playground. (To handle materials like water and sand)

Training:
Every group has to be familiar with the measures and their estimations. In this part they will have the measuring instruments.

After this, every group will do estimations without any measuring instrument:

Cut a string with a length of 1m
Fill a container with 1 l of water
Fill a bag with 1 kg of sand
Do an estimation of 1 minute
Draw an angle of 30º

Finally, they have to do estimations of common objects:

Length/Width ping-pong table
Glass' volume
Dictionary's weight
Time hand dryer is on
Angle formed by olive tree's branches
Wall's height

Students will…

- Use concepts, tools and mathematical strategies to solve problems.
- Maintain a research attitude towards a problem by trying different strategies.
- Use mathematical reasoning in non-mathematical environments.
- Express mathematical ideas with clarity and accuracy and understand those of others.
- Use communication and collaborative work to share and build knowledge based on mathematical ideas.
- Select and use different technologies to manage and display information, and visualize and structure mathematical ideas or processes.

Contents:

- Proportional reasoning.
- Calculation
- Spatial sense and representation of three-dimensional figures.
- Geometric figures, characteristics, properties and construction processes.
- Magnitudes and measurement.
- Metric relations and calculation of measures in figures.

Assessment:

Cooperative work and the classification of the contest will be valued.

Other aspects to emphasize:

The goal of the activity is to make students have an intuitive perception of the most common measures.

RESULTS' TEMPLATE

MESURES	GROUP1	GROUP2	GROUP3	GROUP4
Cut a string with a length of 1m				
Fill a container with 1 l of water				
Fill a bag with 1 kg of sand				
Do an estimation of 1 minute				
Draw an angle of 30º				
Length/Width ping-pong table				
Glass' volume				
Dictionary's weight				
Time hand dryer is on				
Angle formed by olive tree's branches				
Wall's height				

REAL RESULTS AND MARKS DEPENDING ON THE ERRORS' PERCENTAGE

ERROR (%)	±5% 20p	±10% 10p	±20% 5p
Cut a string with a length of 1m	95-105cm	90-110cm	80-120cm
Fill a container with 1 l of water	95-105cl	90-110cl	80-120cl
Fill a bag with 1 kg of sand	950-1050g	900-1100g	800-1200g
Do an estimation of 1 minute	57-63''	54-66''	48-72''
Draw an angle of 30º	28.5-31.5^o	27-33^o	24-36^o
Length/Width ping-pong table (152,5/273)	145-160cm/259-287cm	137-168cm/246-300cm	122-183cm/218-328cm
Glass' volume (450cl)	428-473cl	405-495cl	360-540cl
Dictionary's weight (1325g)	1259-1391g	1192-1458g	1060-1590g
Time hand dryer is on (46'')	44-48''	41-51''	36-56''
Angle formed by olive tree's branches (55^o)	52-58^o	50-60^o	44-66^o
Wall's height (5m)	4.75-5.25m	4.5-5.5m	4-6m

GROUPS' MARKS

MARKS	GROUP1	GROUP2	GROUP3	GROUP4
Cut a string with a length of 1m				
Fill a container with 1 l of water				
Fill a bag with 1 kg of sand				
Do an estimation of 1 minute				
Draw an angle of 30º				
Length/Width ping-pong table				
Glass' volume				
Dictionary's weight				
Time hand dryer is on				
Angle formed by olive tree's branches				
Wall's height				
TOTAL				

STUDENTS' OPINION:

1. Did you enjoy the activity? (from 0 to 10)

2. Do you think that you have learned how to do estimations? (from 0 to 10)

3. Do you think these knowledges will be useful for you in the future? (from 0 to 10)

4. Did you like to work in a group? **(**from 0 to 10)

5. In general, what mark would you give to this activity, taking into account whether you learned or not and if you found it a motivating activity. (from 0 to 10)

6. What positive aspects would you highlight of the activity?

7. What negatives?

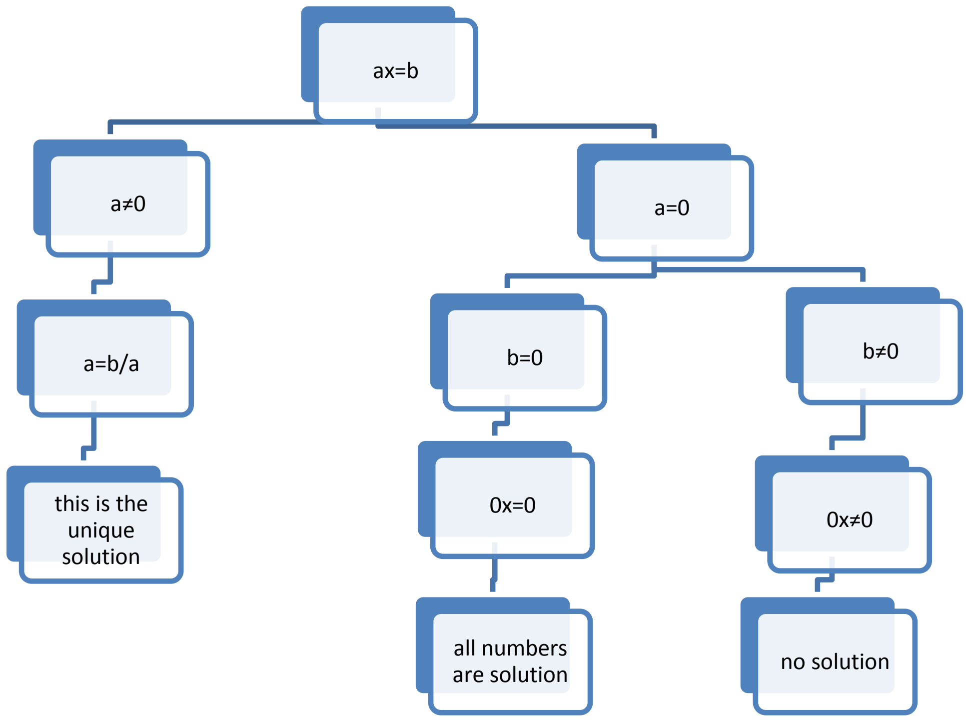
CLASSIFYING EQUATIONS DEPENDING ON THE NUMBER OF SOLUTIONS

Linear equations

When you finish the resolution of a linear equation you have an expression like this:

$ax = b$ where a and b are numbers and x is the unknown.

Next step, as you know is to isolate x : $x = b/a$, this step is only possible if a is not zero.



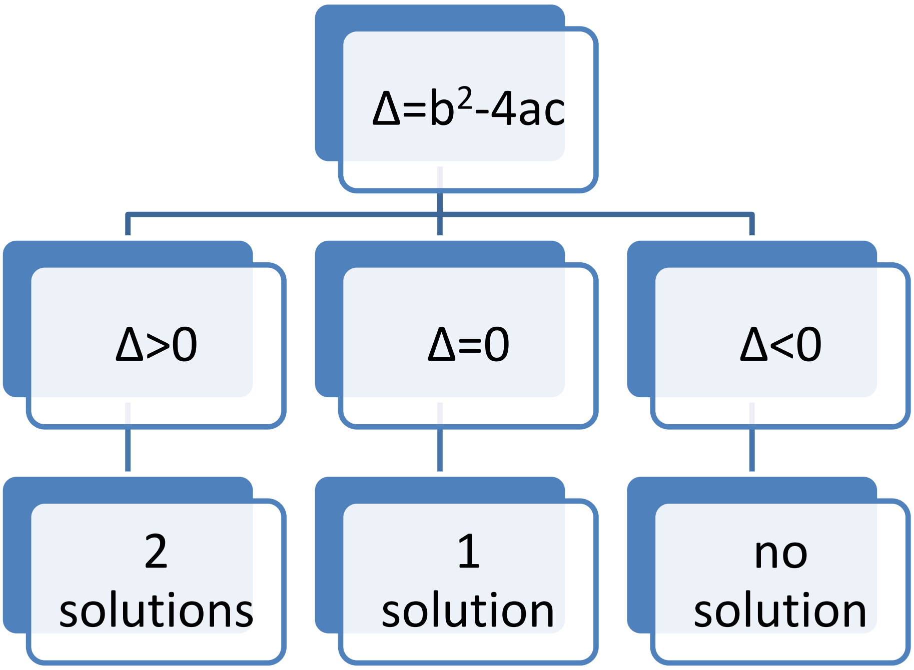
Second degree equations

All second degree equations ($ax^2+bx+c=0$) can be solved with the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression inside de root is called discriminant and usually is represented by a capital Greek letter called Delta: Δ, so: $\Delta = b^2 - 4ac$.

Depending on Δ the root will have 0, 1 or 2 solutions and this is the criteria we must follow to classify the equation:



ACTIVITY 1

<table>
<tr><td rowspan="2">Linear equations (ax=b)</td><td>Have 2 solutions</td><td rowspan="5">when</td><td rowspan="2">Δ</td><td>Is positive</td><td rowspan="5">b</td><td rowspan="2">Is zero</td></tr>
<tr><td>Have 1 solution</td><td>Is negative</td></tr>
<tr><td rowspan="3">Second degree equations</td><td rowspan="2">Haven't solution</td><td rowspan="3">a</td><td>Is zero</td><td rowspan="3">Is not zero</td></tr>
<tr><td>Is not zero</td></tr>
<tr><td>Have ∞ solutions</td><td>Is zero and</td></tr>
</table>

Write 6 sentences to describe the number of solutions that linear and second degree equations could have (choose one item of each box, maybe you don't need to choose one of all of them).

1.

2.

3.

4.

5.

6.

ACTIVITY 2

a) How many solutions have the following equations? (You don't need to solve them, just calculate their discriminants)

- $x^2 + x + 1 = 0$

Δ = It has ………………….solution/s

- $2x^2 + 3x – 7 = 0$

Δ = ……………………………………………….

- $x^2 – 10x + 25 = 0$

Δ = ……………………………………………….

b) Study the number of solutions of the equation: $x^2 + kx + 4 = 0$ depending on the values of letter k.

<u>Solution: (write sentences like this one to give the solution)</u>

- When k ……………………………the equation has ……………………………solution/s.
- ……
- ……

ACTIVITY 3

Organize the following flashcards about how to solve second degree equations. Then write the steps you need to follow to solve them.
(Some help will be provided)

-Second degree equations can be written……………………………………………………………………..

COMPLETE EQUATIONS

………………………………………………………………….the equation is called complete.

To solve them……..

The number of solutions depends on……………………………………………………………………………..

INCOMPLETE EQUATIONS

……..

……..

……..

Key words: (discriminant, isolate, common factor, solve, square root, positive, negative, have to, must, should, greater than, less than…)

$ax^2+bx+c = 0$	a, b, c $a \neq 0$	$b \neq 0$ $c \neq 0$
$b=0$	$c=0$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$ax^2+bx=0$	$ax^2+c=0$	$x(ax+b)=0$
$ax^2=-c$	$x=0$ $ax+b=0$	$x^2= \frac{-c}{a}$
$x=0$ $x=\frac{-b}{a}$	$x= +\sqrt{\frac{-c}{a}}$ $x= -\sqrt{\frac{-c}{a}}$	$\frac{-c}{a} < 0$
$\frac{-c}{a} > 0$	$\frac{-c}{a} = 0$	$\Delta=b^2-4ac=0$
$\Delta=b^2-4ac>0$	$\Delta=b^2-4ac<0$	one solution
one solution	two solutions	two solutions
two solutions	no solution	no solution

Potential Project Roles

- **Maths expert:** This student is in charge of organizing the final product of the project: a presentation. A good level of maths is required for this role.

- **Secretary and language expert:** This person takes notes whenever the group meets and distributes these notes to the rest of the group highlighting sections relevant for their parts of the project. This is the main responsible person of linguistic correction of the final production. To be a well-organized person and to have a good English level is required for this role.

- **ITC expert:** This person has to make a power point presentation. A good level of computer skills is required for this role.

- **Public relations expert**: This person has to present to the rest of the class the power point. A good level of spoken English and to be a dynamic person are good qualities for this role.

Assigning roles

Discuss with the members of your group which expert fits the best for you.

1. Every group member has to write a brief explanation of the role preferred.

You can use the following ideas:

Wish:

I would like to be...

I prefer to be…

I want to be…

Qualities:

Because:

I've got a good level of...

I am a … person

I know how to…

Evidences:

My marks in…………………………………are…………………..

Other evidences you want to share with your group.

2. Every group member has to explain to the others his preferences.

3. If a role has been chosen by more than one person then organize a voting and decide.

Before reading:

Answer these questions:

- What do you know about the topic? Previous knowledge.

- What will you learn? Make a prediction.

During reading:

Make a list of the words you don't know.

What kind of equations or systems does your text talk about?

How you can identify them? Explain it.

How can you solve them? Explain it.

After reading: (Sharing your information)

Working all together,

- Try to translate all the words that your team don't know.
- Write a comprehensible and short text explaining what kind of equations or systems you have studied.
- Write instructions to solve your equations or systems.
- Make a list of exercises for your classmates.

Final project:

- Make a power point presentation using all your work.
- You will be teachers for a day and you will explain the lesson and help your classmates to solve their doubts about the exercises you have prepared with your team.

PROJECT'S TEXTS

- **Simultaneous non-linear equations**
- **Radical equations**
- **Rational equations**
- **Biquadratic equations**
- **Polynomial long division**

Source: Esobook Mathematics 4ESo

5.2 Simultaneous non-linear equations

Simultaneous equations are "non-linear" if at least one equation is non-linear.

We normally solve non-linear simultaneous equations by substitution:

- In the simplest equation we isolate the variable that is easiest to isolate.
- We substitute the value of that variable in the other equation.

Example 1: Solve the following simultaneous equations $\begin{cases} y + 2x - 3 = 0 \\ y + x^2 = 6 \end{cases}$

In the first equation we isolate one of the unknowns, for example y: $y = 3 - 2x$

Then we substitute that expression in the second equation:

$$(3 - 2x) + x^2 = 6 \quad \rightarrow \quad x^2 - 2x - 3 = 0$$

It is a quadratic equation, so we use the quadratic formula:

$$x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \mathbf{3}\ or\ \mathbf{-1}$$

3 and -1 are the solutions for x. For each of these values we find the corresponding value of the other unknown (y). We substitute each value in one of the given equations (use the lowest degree equation) and solve for the unknown (In this case we substitute each x-value in the first equation):

$\boxed{x = 3} \rightarrow y + 6 - 3 = 0 \rightarrow y = -3$ $\qquad$ $\boxed{x = -1} \rightarrow y - 2 - 3 = 0 \rightarrow y = 5$

Finally, the solutions to the given simultaneous equations are (3,-3) and (-1,5).

Example 2: Solve the following simultaneous equations $\begin{cases} 2y - x = 1 \\ xy + x^2 - 26 = 0 \end{cases}$

In the first equation we isolate x: $x = 2y - 1$ If we substitute that expression in the second equation, we get:

$$(2y-1)y + (2y-1)^2 - 26 = 0 \quad \rightarrow \quad 2y^2 - y + 4y^2 + 1 - 4y - 26 = 0$$

And simplifying that expression we have a quadratic equation: $6y^2 - 5y - 25 = 0$
The solutions are:

$$X = \frac{5 \pm \sqrt{25+600}}{12} = \frac{5 \pm 25}{12} = \mathbf{5/2} \text{ and } \mathbf{-5/3}$$

Using the first equation we find that the solutions are (4, 5/2) and (-13/3, -5/3).

11 Solve these simultaneous non-linear equations:

a) $\begin{cases} 4x^2 + y^2 = 13 \\ x + y = 2 \end{cases}$ b) $\begin{cases} x = y^2 \\ x^2 + y^2 = 6 \end{cases}$ c) $\begin{cases} x^2 + 4y^2 = 16 \\ x^2 + y^2 = 9 \end{cases}$ d) $\begin{cases} y = x^2 \\ -x^2 + 2x = y \end{cases}$

Source: https://www.mcckc.edu/tutoring/docs/br/math/equat_inequ/Solving_Radical_Equations.pdf

Solving Radical Equations

Index or Root → $\sqrt[n]{ax+b} = c$ ← Radicand

Solving equations requires isolation of the variable. Equations that contain a **variable inside of a radical** require algebraic manipulation of the equation so that the variable "comes out" from underneath the radical(s). This can be accomplished by **raising both sides of the equation to the "nth" power**, where **n is the "index" or "root"** of the radical. When the index is a 2 (*i.e. a square root*), we call this method "*squaring both sides*." Sometimes the equation may contain more than one radical expression, and it is possible that the method may need to be used more than once to solve it.

When the index is an even number *(n = 2, 4, etc.)* this method can introduce **extraneous solutions**, so it is necessary to verify that any answers obtained actually work. This can be accomplished by plugging the answer(s) back in to the original equation to see if the resulting values satisfy the equation. It is also good practice to check the solutions when there is an odd index to identify any algebra mistakes.

General Solution Steps:

Step 1. Isolate the Radical(s) and identify the index *(n).*
Step 2. Raise both sides of the equation to the "nth" power.
Step 3. Use algebraic techniques *(i.e. factoring, combining like terms,…)* to isolate the variable.
Repeat Steps 1 and 2 if necessary.
Step 4. Check answers. Eliminate any extraneous solutions from the final answer.

Examples:

a. $\boldsymbol{\sqrt{5-x}-3=0}$ *(Problem with 1 radical)*

Step 1: Isolate the Radical $\sqrt{5-x}=3$

Step 2: Square both Sides $\left(\sqrt{5-x}\right)^2 = (3)^2$

Step 3: Solve for "x": $5-x=9$

$-x=4$

$\boldsymbol{x=-4}$

Step 4: Check Answers

$\sqrt{5-(-4)}-3=0$

$\sqrt{9}-3=0$

$3-3=0$

$0=0$ ✓

b. $\boldsymbol{\sqrt{x^2-2}-\sqrt{x+4}}=0$ *(Problem with 2 radicals **and** no other "non-zero" terms)*

Step 1: Isolate the radicals $\sqrt{x^2-2}=\sqrt{x+4}$

Step 2: Square both Sides $\left(\sqrt{x^2-2}\right)^2 = \left(\sqrt{x+4}\right)^2$

Step 3: Solve for "x": $x^2-2=x+4$

$x^2-x-6=0$

$(x+2)(x-3)=0$

$\boldsymbol{x=-2 \quad and \quad x=3}$

Step 4: Check Answers →

$\sqrt{(-2)^2-2}-\sqrt{(-2)+4}=0$ | $\sqrt{(3)^2-2}-\sqrt{(3)+4}=0$
$\sqrt{4-2}-\sqrt{2}=0$ | $\sqrt{9-2}-\sqrt{7}=0$
$\sqrt{2}-\sqrt{2}=0$ | $\sqrt{7}-\sqrt{7}=0$
$0=0$ ✓ | $0=0$ ✓

c. $\sqrt{z-6}-\sqrt{z+9}+3=0$ *(Problem with 2 radicals **and** another "non-zero" term)*

Step 1: Isolate the radicals so that they are on opposite sides of the " = " sign

$$\sqrt{z-6}+3=\sqrt{z+9}$$

Step 2: Square both Sides $\left(\sqrt{z-6}+3\right)^2=\left(\sqrt{z+9}\right)^2$

This term will need to be "FOIL-ed"

Step 3: Solve for "x". *Because a* *<u>radical still remains</u>* *during this process,* *<u>repeat Steps 1 and 2</u>*.

FOIL → $(\sqrt{z-6})\cdot(\sqrt{z-6})+3\sqrt{z-6}+3\sqrt{z-6}+(3)(3)=z+9$

Combine like terms → $(z-6)+6\sqrt{z-6}+9=z+9$

$z+3+6\sqrt{z-6}=z+9$

Radical still remains → $6\sqrt{z-6}=6$

(***Repeat* Step 1**) $\sqrt{z-6}=1$

(***Repeat* Step 2**) $\left(\sqrt{z-6}\right)^2=(1)^2$

$z-6=1$

$\boldsymbol{z=7}$

Step 4: Check Answer

$\sqrt{7-6}-\sqrt{7+9}+3=0$

$\sqrt{1}-\sqrt{16}+3=0$

$1-4+3=0$

$0=0$ ✓

d. $4+\sqrt[3]{x-6}=2$ *(Problem with an "nth" root)*

Step 1: Isolate the radical $\sqrt[3]{x-6}=-2$

Step 2: Raise both sides to the "nth" power *(n = 3)*

$$\left(\sqrt[3]{x-6}\right)^3=(-2)^3$$

Step 3: Solve for "x": $x-6=-8$

$\boldsymbol{x=-2}$

Step 4: Check Answer

$\sqrt[3]{(-2)-6}=-2$

$\sqrt[3]{-8}=-2$

$\sqrt[3]{(-2)(-2)(-2)}=-2$

$-2=-2$ ✓

Radical equations can also be re-written in their exponential form and then both sides raised to the reciprocal power. This method can be more efficient if the radicand is raised to a power as in the example below. *(Note: the reciprocal power can be found by flipping the fractional power upside down.)*

e. $\sqrt[3]{(x-2)^2} = 4$ *(Problem with a radicand that is raised to a power)*

Option 1: ("nth" power)

Step 1: Isolate the radical $\sqrt[3]{(x-2)^2} = 4$

Step 2: Raise both sides to the *nth* power

$$\left(\sqrt[3]{(x-2)^2}\right)^3 = (4)^3$$

Step 3: Solve for "x": $(x-2)^2 = 64$

$$x - 2 = \pm\sqrt{64}$$

$$x = 2 \pm 8$$

$$\boldsymbol{x = -6 \ \ and \ \ 10}$$

Step 4: A check shows that both solutions work in the original equation.

Option 2: (reciprocal power)

Step 1: Isolate & Re-Write in Exponential Form

$$(x-2)^{\frac{2}{3}} = 4$$

Step 2: Raise both sides to the *reciprocal* power

$$\left[(x-2)^{\frac{2}{3}}\right]^{\frac{3}{2}} = [4]^{\frac{3}{2}}$$

$$(x-2)^1 = \sqrt[2]{4^3}$$

Step 3: Solve for "x" $x - 2 = \pm\sqrt{64}$

$$x = 2 \pm 8$$

$$\boldsymbol{x = -6 \ \ and \ \ 10}$$

Radical Equations Practice Problems

	Problem	**Answer**
1.	$\sqrt{x+1} = 4$	$x = 15$
2.	$\sqrt{2t+15} = t$	$t = 5 \quad (Note:\ t = -3\ is\ Extraneous)$
3.	$\sqrt{p-4} = -5$	No Solution
4.	$\sqrt{2-x} = x-2$	$x = 2 \quad (Note:\ x = 1\ is\ Extraneous)$
5.	$\sqrt{s^2-1} = \sqrt{5s-5}$	$s = 1,\ 4$
6.	$\sqrt[3]{x} = 4$	$x = 64$
7.	$\sqrt[4]{2p+2} = 3$	$p = 39$
8.	$\sqrt[3]{x+1} = \sqrt[3]{x^2-5}$	$x = -2,\ 3$
9.	$\sqrt{2-\sqrt{x}} = \sqrt{x}$	$x = 1 \quad (Note:\ x = 4\ is\ Extraneous)$
10.	$\sqrt{3t+1} + \sqrt{5-t} = 4$	$t = 1,\ 5$
11.	$\sqrt{8-p} = 2 + \sqrt{2p+3}$	$p = -1 \quad (Note:\ p = \frac{47}{9}\ is\ Extraneous)$
12.	$\sqrt[3]{4x^2+2x-8} = x$	$x = -\sqrt{2},\ \sqrt{2},\ 4$
13.	$\sqrt[3]{(x-4)^2} = 1$	$x = 3,\ 5$
14.	$\sqrt{(x+1)^3} = 9$	$x = -1 + 3\sqrt[3]{3}$
15.	$\sqrt{s^3} = -4$	No Solution

Source: http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U11_L2_T1_text_container.html

Solving Rational Equations

Learning Objective(s)

- Solve rational equations using the techniques for simplifying and manipulating rational expressions.

Introduction

Equations that contain **rational expressions** are called **rational equations**. We can solve these equations using the techniques for performing operations with rational expressions and for solving algebraic equations.

Solving Rational Equations Using Common Denominators

One method for solving rational equations is to rewrite the rational expressions in terms of a common denominator. Then, since we know the numerators are equal, we can solve for the variable. To illustrate this, let's look at a very simple equation:

$$\frac{3}{4}=\frac{x}{4}$$

$x = 3$

Since the denominator of each expression is the same, the numerators must be equivalent as well. This means that $x = 3$.

This is true for rational equations with polynomials too:

$$\frac{2x-5}{x-4}=\frac{11}{x-4}$$

$2x - 5 = 11$

$x = 8$

Again, since the denominators are the same, we know the numerators must also be equal. So we can set them equal to one another and solve for *x*.

We should check our solution in the original rational expression:

$$\frac{2x-5}{x-4}=\frac{11}{x-4}$$

$$\frac{2(8)-5}{8-4}=\frac{11}{8-4}$$

$$\frac{11}{4}=\frac{11}{4}$$

The solution checks, and since $x = 8$ does not result in division by 0, the solution is valid.

When the terms in a rational equation have unlike denominators, solving the equation will involve some extra work. Here's an example:

Example	
Problem **Solve the equation** $\frac{x+2}{8}=\frac{3}{4}$	
	There are no excluded values because the denominators are both constants.
$\frac{x+2}{8}=\frac{3}{4}\bullet\frac{2}{2}$	Find a common denominator and rewrite each expression with that denominator.
$\frac{x+2}{8}=\frac{6}{8}$	The common denominator is 8.
$x+2=6$	Since the denominators are the same, the numerators must be equal for the equation to be true. Solve for *x*.

$x = 4$	
$\frac{4+2}{8} = \frac{3}{4}$	Check the solution by substituting 4 for *x* in the original equation.
$\frac{6}{8} = \frac{3}{4}$	
$\frac{3}{4} = \frac{3}{4}$	
Answer	
$x = 4$	

Another way of solving rational equations is to multiply both sides of the equation by the common denominator. This eliminates the denominators and turns the rational equation into a polynomial equation. Here is the same equation we just solved:

Example	
Problem **Solve the equation** $\frac{x+2}{8} = \frac{3}{4}$	
	There are no excluded values because the denominators are both constants.
$\frac{x+2}{8} \bullet 8 = \frac{3}{4} \bullet 8$	Multiply both sides by the least common denominator
$x+2 = \frac{24}{4}$	Simplify
$x + 2 = 6$	
$x + 2 - 2 = 6 - 2$	Solve for *x*
$x = 4$	

Answer

$x = 4$

Now that we understand the techniques, let's look at an example that has variables in the denominator too. Remember that whenever there are variables in the denominator, we need to find any values that are excluded from the **domain** because they'd make the denominator zero.

To solve this equation, we can multiply both sides by the least common denominator:

Example		
Problem	**Solve** $\frac{7}{x+2}+\frac{5}{x-2}=\frac{10x-2}{x^2-4}$	
	$x + 2 = 0$ $x = -2$ $x - 2 = 0$ $x = 2$ $(x + 2)(x - 2) = 0$ $x = -2, 2$	First determine the **excluded values**. These are the values of x that result in a 0 denominator.
	denominators: $x + 2$ $x - 2$ $x^2 - 4 = (x - 2)(x + 2)$	Find the common denominator of $x - 2$, $x + 2$, and $x^2 - 4$ Since $(x - 2)$ and $(x + 2)$ are both factors of $x^2 - 4$, the least common denominator is $(x - 2)(x + 2)$ or $x^2 - 4$
	least common denominator: $(x - 2)(x + 2)$	

$(x+2)(x-2)\left(\dfrac{7}{x+2}+\dfrac{5}{x-2}\right)=\dfrac{10x-2}{x^2-4}\bullet(x+2)(x-2)$	Multiply both sides of the equation by the common denominator.
$\dfrac{7(x+2)(x-2)}{(x+2)}+\dfrac{5(x+2)(x-2)}{(x-2)}=\dfrac{(10x-2)}{(x^2-4)}\bullet(x^2-4)$	
$7(x-2)+5(x+2)=10x-2$	Simplify
$7x - 14 + 5x + 10 = 10x - 2$	
$12x - 4 = 10x - 2$ $12x - 10x - 4 = 10x - 10x - 2$	Solve for x
$2x - 4 = -2$	
$2x - 4 + 4 = -2 + 4$	
$2x = 2$	
$x = 1$	Check to be sure that the solution is not an excluded value. (It is not.)
$\dfrac{7}{x+2}+\dfrac{5}{x-2}=\dfrac{10x-2}{x^2-4}$	Check the solution in the original equation.
$\dfrac{7}{1+2}+\dfrac{5}{1-2}=\dfrac{10\bullet 1-2}{1^2-4}$	

$$\frac{7}{3}+\frac{5}{-1}=\frac{10-2}{1-4}$$

$$\frac{7}{3}-\frac{15}{3}=\frac{8}{-3}$$

$$-\frac{8}{3}=-\frac{8}{3}$$

Answer $x = 1$

Solve the equation $\frac{4}{m}=\frac{3}{m-2}$, $m \neq$ 0 or 2

A) $m = 2$

B) no solution

C) $m = 8$

Show/Hide Answer

We've seen that there is more than one way to solve rational equations. Because both of these techniques manipulate and rewrite terms, sometimes they can produce solutions that don't work in the original form of the equation. These types of answers are called **extraneous solutions**. These solutions are artifacts of the solving process and not real answers at all. That's why we should always check solutions in the original equations—we may find that they yield untrue statements or produce undefined expressions.

Solve the equation:

$$\frac{1}{x-6}+\frac{x}{x-2}=\frac{4}{x^2-8x+12}$$

A) $x = -1$

B) x = -1, 6

C) x = -4, 3

D) no solution

Show/Hide Answer

Summary

We solve rational equations by finding a common denominator. We can then follow either of two methods. We can rewrite the equation so that all terms have the common denominator and we can solve for the variable with just the numerators. Or we can multiply both sides of the equation by the common denominator so that all terms become polynomials instead of rational expressions.

An important step in solving rational equations is to reject any extraneous solutions from the final answer. Extraneous solutions are solutions that don't satisfy the original form of the equation because they produce untrue statements or are excluded values that make a denominator equal to 0.

Source: https://www.sangakoo.com/en/unit/biquadratic-equations

Biquadratic equations

We are going to learn how to solve equations of this type:

$$ax^4 + bx^2 + c = 0$$

that is, 4-degree equations in which we do not have terms of an odd degree. These equations are called biquadratic.

To solve them we will convert them into quadratic equations.

Let's see an example that will help us to better understand the process:

Example

We want to solve the following equation:

$$x^4 - 8x^2 + 12 = 0$$

If we change the variable $x^2 = t$, we get the equation:

$$t^2 - 8t + 12 = 0$$

This equation can be solved:

$$t = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} =$$
$$= \begin{cases} t_1 = \frac{8+4}{2} = \frac{12}{2} = 6 \\ t_2 = \frac{8-4}{2} = \frac{4}{2} = 2 \end{cases}$$

Therefore we have two solutions:

$$t_1 = 6$$
$$t_2 = 2$$

But we want to find the value of x; if we undo the first change we will have:

$$\begin{array}{rcl} x^2 & = & t & \longrightarrow & x & = & \pm\sqrt{t} \\ x & = & \pm\sqrt{t_1} & \longrightarrow & x & = & \pm\sqrt{6} \\ x & = & \pm\sqrt{t_2} & \longrightarrow & x & = & \pm\sqrt{2} \end{array}$$

Therefore we obtain 4 solutions:

$$x_1 = \sqrt{6} \quad x_3 = \sqrt{3}$$
$$x_2 = -\sqrt{6} \quad x_4 = -\sqrt{2}$$

Now that we have seen an example of how to solve this type of equations, we could wonder if we will always obtain 4 solutions.

The answer is no, and let's see why.

The number of solutions of the equation will depend on the number of solutions of the quadratic equation since for every positive solution of the quadratic equation we will have 2 solutions in the biquadratic one.

This way we can make sure that we will not have more than 4 solutions in the biquadratic equation.

Source: https://www.math.ucdavis.edu

POLYNOMIAL LONG DIVISION

Polynomial long division is normal long division but with polynomials instead of just numbers. It acts in exactly the same ways that our normal quotients of numbers do. To start with, let's review the process of our usual long division. We will then extend to polynomials.

Problem 1: Use long division to divide 7 into 323.

Answer:

$$\begin{array}{r} 46 \\ 7\overline{)323} \\ \underline{280} \\ 43 \\ \underline{42} \\ 1 \end{array}$$

Here we start with the set up $7\overline{)323}$. First we ask how many times 7 goes into 3 because 3 is the leading number on the left. Well...it doesn't, so we move over a decimal place. Now we ask how many times 7 goes into 32. We know $7 \cdot 4 = 28$, so we throw the 4 on top above the 2. Then we actually multiply the 4 times 7 and write it underneath the first two terms. We then subtract this 28 from the 32 and get 4. Next we just bring the final right hand 3 down to make the 43.

Moving to the next decimal place, we ask how many times 7 goes into the new dividend 43. We know $7 \cdot 6 = 42$, so we can put the 6 on top and the 42 under the 43 and subtract again. Doing this leaves us with 1. Since 7 does not divide into 1, we are done. This means that $323 = 46 \cdot 7 + 1$ where we call 46 the quotient (which the whole number of times 7 can fit into 323), and the remainder 1 (the amount left over after 7 has gone in as many times as possible).

It turns out polynomial long division is very similar. The algorithm is exactly the same, we just have powers of x to take care of (along with their coefficients).

Problem 2: Use Polynomial long division to divide $x - 1$ into $x^4 + x^3 - 3x^2 + 3x - 2$.

Answer: We set everything up just like last time. Start with

$$x - 1\overline{)x^4 + x^3 - 3x^2 + 3x - 2}$$

Now we ask the same question every time: what would you multiply x by to get ____?

Date: September 22, 2015 By Tynan Lazarus.

POLYNOMIAL LONG DIVISION

- **Step 1:** What would you multiply x by to get x^4? Of course we would answer x^3. So, put the x^3 on top:

$$\begin{array}{r|l} & x^3 \\ \hline x-1 & x^4 + x^3 - 3x^2 + 3x - 2 \end{array}$$

- **Step 2:** Multiply the x^3 across both the terms of the divisor and place them under the corresponding powers of x and subtract. Then bring down the next term just like in normal long division:

$$\begin{array}{r|l} & x^3 \\ \hline x-1 & x^4 + x^3 - 3x^2 + 3x - 2 \\ & -x^4 + x^3 \\ \hline & \quad 2x^3 - 3x^2 \end{array}$$

- **Step 3:** What would you multiply x by to get $2x^3$? We would answer $2x^2$, so we add that to the top:

$$\begin{array}{r|l} & x^3 + 2x^2 \\ \hline x-1 & x^4 + x^3 - 3x^2 + 3x - 2 \\ & -x^4 + x^3 \\ \hline & \quad 2x^3 - 3x^2 \end{array}$$

- **Step 4:** Now take the $2x^2$ and multiply across both of the terms of the divisor and place them under the corresponding powers of x and subtract. Bring down the $3x$ to get:

$$\begin{array}{r|l} & x^3 + 2x^2 \\ \hline x-1 & x^4 + x^3 - 3x^2 + 3x - 2 \\ & -x^4 + x^3 \\ \hline & \quad 2x^3 - 3x^2 \\ & \quad -2x^3 + 2x^2 \\ \hline & \qquad -x^2 + 3x \end{array}$$

- **Step 5:** What would you multiply x by to get $-x^2$? We would answer $-x$, so we add that to the top:

$$\begin{array}{r|l} & x^3 + 2x^2 - x \\ \hline x-1 & x^4 + x^3 - 3x^2 + 3x - 2 \\ & -x^4 + x^3 \\ \hline & \quad 2x^3 - 3x^2 \\ & \quad -2x^3 + 2x^2 \\ \hline & \qquad -x^2 + 3x \end{array}$$

- **Step 6:** Now take the $-x$ and multiply across both of the terms of the divisor and place them under the corresponding powers of x and subtract. Bring down the -2 to get:

$$\begin{array}{rrrrrr}
 & & x^3 & +2x^2 & -x & \\
\cline{2-6}
x-1) & x^4 & +x^3 & -3x^2 & +3x & -2 \\
 & -x^4 & +x^3 & & & \\
\cline{2-3}
 & & 2x^3 & -3x^2 & & \\
 & & -2x^3 & +2x^2 & & \\
\cline{3-4}
 & & & -x^2 & +3x & \\
 & & & x^2 & -x & \\
\cline{4-5}
 & & & & 2x & -2
\end{array}$$

- **Step 7:** Finally, what would you multiply x by to get $2x$? Of course just 2, so we add that to the top:

$$\begin{array}{rrrrrr}
 & & x^3 & +2x^2 & -x & +2 \\
\cline{2-6}
x-1) & x^4 & +x^3 & -3x^2 & +3x & -2 \\
 & -x^4 & +x^3 & & & \\
\cline{2-3}
 & & 2x^3 & -3x^2 & & \\
 & & -2x^3 & +2x^2 & & \\
\cline{3-4}
 & & & -x^2 & +3x & \\
 & & & x^2 & -x & \\
\cline{4-5}
 & & & & 2x & -2
\end{array}$$

- **Step 8:** Now take the 2 and multiply across both of the terms of the divisor and place them under the corresponding powers of x and subtract. So we get:

$$\begin{array}{rrrrrr}
 & & x^3 & +2x^2 & -x & +2 \\
\cline{2-6}
x-1) & x^4 & +x^3 & -3x^2 & +3x & -2 \\
 & -x^4 & +x^3 & & & \\
\cline{2-3}
 & & 2x^3 & -3x^2 & & \\
 & & -2x^3 & +2x^2 & & \\
\cline{3-4}
 & & & -x^2 & +3x & \\
 & & & x^2 & -x & \\
\cline{4-5}
 & & & & 2x & -2 \\
 & & & & -2x & +2 \\
\cline{5-6}
 & & & & & 0
\end{array}$$

So, $x^4 + x^3 - 3x^2 + 3x - 2 = (x-1)(x^3 + 2x^2 - x + 2)$. In other words, $x-1$ divides into the polynomial $x^4+x^3-3x^2+3x-2$ "x^3+2x^2-x+2 times".

The next example deals with a remainder.
Problem 3: Compute the quotient of $3x^4 - 5x^3 + 7x^2 - 9x + 3$ by $x^2 + 2$.
Answer: We set everything up as usual. Start with

$$x^2 + 2\overline{)3x^4 - 5x^3 + 7x^2 - 9x + 3}$$

- **Step 1:** What would you multiply the leading term of the divisor, x^2, by to get the leading term of the dividend$3x^4$? We would answer $3x^2$. So, put the $3x^2$ on top:

$$\begin{array}{r|rrrrr} & & & 3x^2 & & \\ \hline x^2+2) & 3x^4 & -5x^3 & +7x^2 & -9x & +3 \end{array}$$

- **Step 2:** Multiply the $3x^2$ across both the terms of the divisor and place them under the corresponding powers of x and subtract. Then bring down the next term just like in normal long division. Notice that we get something a little different this time. But different isn't bad, we can just continue as normal:

$$\begin{array}{r|rrrrr} & & & 3x^2 & & \\ \hline x^2+2) & 3x^4 & -5x^3 & +7x^2 & -9x & +3 \\ & -3x^4 & & -6x^2 & & \\ \hline & & -5x^3 & +x^2 & -9x & \end{array}$$

- **Step 3:** What would you multiply x^2 by to get $-5x^3$? We would answer $-5x$, so we add that to the top:

$$\begin{array}{r|rrrrr} & & & 3x^2 & -5x & \\ \hline x^2+2) & 3x^4 & -5x^3 & +7x^2 & -9x & +3 \\ & -3x^4 & & -6x^2 & & \\ \hline & & -5x^3 & +x^2 & -9x & \end{array}$$

- **Step 4:** Now take the $-5x$ and multiply across both of the terms of the divisor and place them under the corresponding powers of x and subtract. Bring down the 3 to get:

$$\begin{array}{r|rrrrr} & & & 3x^2 & -5x & \\ \hline x^2+2) & 3x^4 & -5x^3 & +7x^2 & -9x & +3 \\ & -3x^4 & & -6x^2 & & \\ \hline & & -5x^3 & +x^2 & -9x & \\ & & 5x^3 & & +10x & \\ \hline & & & x^2 & +x & +3 \end{array}$$

- **Step 5:** What would you multiply x^2 by to get x^2? It sounds stupid, but it still makes sense so we'll answer it. We'd have to multiply by 1, so we add that to the top:

$$
\begin{array}{r|l}
 & 3x^2 \quad - 5x + 1 \\
\hline
x^2 + 2) & 3x^4 - 5x^3 + 7x^2 - 9x + 3 \\
 & -3x^4 \qquad\quad - 6x^2 \\
\hline
 & \qquad - 5x^3 + x^2 - 9x \\
 & \qquad\ \ 5x^3 \qquad + 10x \\
\hline
 & \qquad\qquad\quad x^2 + x + 3
\end{array}
$$

- **Step 6:** Now take the 1 and multiply across both of the terms of the divisor and place them under the corresponding powers of x and subtract:

$$
\begin{array}{r|l}
 & 3x^2 \quad - 5x + 1 \\
\hline
x^2 + 2) & 3x^4 - 5x^3 + 7x^2 - 9x + 3 \\
 & -3x^4 \qquad\quad - 6x^2 \\
\hline
 & \qquad - 5x^3 + x^2 - 9x \\
 & \qquad\ \ 5x^3 \qquad + 10x \\
\hline
 & \qquad\qquad\quad x^2 + x + 3 \\
 & \qquad\qquad - x^2 \qquad - 2 \\
\hline
 & \qquad\qquad\qquad\quad x + 1
\end{array}
$$

- **Step 7:** What would we multiply x^2 by to get x? Now that is a weird question. Notice we would have to multiply by $\frac{1}{x}$, which is not in the same pattern we've been doing. So we stop here. Essentially we are only multiplying by nonnegative powers of x. If we ever hit a negative power of x we know we are done. But there's still that $x+1$ sitting down there. Just like the 1 that was left over in problem 1, this is the remainder for this problem.

  So, $3x^4 - 5x^3 + 7x^2 - 9x + 3 = (x^2 + 2)(3x^2 - 5x + 1) + x + 1$. In other words, we get that $x^2 + 2$ divides into $3x^4 - 5x^3 + 7x^2 - 9x + 3$ a total "number" of $3x^2 - 5x + 1$ times with a remainder of $x + 1$.

The Theory

Ok, where the heck is this stuff coming from? Let's start with the normal long division. Division really comes from the process to satisfy the following theorem.

Theorem (The Division Algorithm): For any integers a, b with $b \neq 0$, there exist **unique** integers q and r such that $a = bq + r$ where $0 \leq r < |b|$. The integer q is called the quotient and r is the remainder.

The process of long division is how we find the q and r to put in the equation. When we do polynomial long division, it is almost identical but we use functions instead (it even goes by the same name).

Theorem (Division Algorithm): Let D be an integral domain, and $a(x), b(x)$ be in $D[x]$ such that the leading coefficients of $a(x)$ is invertible in D. Then there exist polynomials $q(x), r(x)$ in $D[x]$ such that

$$b(x) = a(x)q(x) + r(x)$$

where $r(x) = 0$ or has a degree less than that of $a(x)$. Furthermore, polynomials $q(x)$ and $r(x)$ are unique.

Don't worry about the $D[x]$ and invertible stuff, it just means we are working with polynomials and you can divide by them. What we get out of this is that it looks identical to the usual case with numbers, and that the process of polynomial long divison gives us the quotient $q(x)$ and remainder $r(x)$.